

MA261 Quiz 3

June 24, 2016

Problem 1.

Given the acceleration of a particle $\mathbf{a}(t) = \langle 2, 6t, 4 \rangle$, the initial velocity $\mathbf{v}(0) = \langle -1, 0, -3 \rangle$ and the initial position $\mathbf{r}(0) = \langle 0, -2, 0 \rangle$, find the position of this particle at the time $t = 2$.

Solution.

$$\begin{aligned} \mathbf{r}(t) &= \langle t^2 - t, t^3 - 2, 2t^2 - 3t \rangle \\ \mathbf{r}(2) &= \langle 2, 6, 2 \rangle \end{aligned}$$

Problem 2.

Find the length of the given curve

$$\begin{aligned} \mathbf{r}(t) &= \left\langle \frac{1}{3}t^3 - t, 2t \sin t + 2 \cos t, 2t \cos t - 2 \sin t \right\rangle \\ 0 &\leq t \leq 1 \end{aligned}$$

Solution.

$$\mathbf{r}'(t) = \langle t^2 - 1, 2t \cos t, -2t \sin t \rangle$$

$$\begin{aligned} l &= \int_0^1 \sqrt{(t^2 - 1)^2 + (2t \cos t)^2 + (-2t \sin t)^2} dt \\ &= \int_0^1 \sqrt{t^4 + 2t^2 + 1} dt \\ &= \int_0^1 (t^2 + 1) dt = 4/3 \end{aligned}$$